OPTIMAL SYNTHESIS OF MULTIMODE WAVEGUIDE COMPONENTS

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Abstract. This paper describes realization of metal waveguide component synthesis procedure based on combination of the optimal iterative synthesis principles and the full wave analysis by electric field integral equation (EFIE). The synthesis procedure takes two electromagnetic field distributions (from input to output and backward) per each iteration and operates with tangential magnetic and normal electric fields on the waveguide wall, that are simply expressed in terms of surface current calculated in EFIE solution. The EFIE approach allows one to calculate all components of electromagnetic fields (EFIE is derived directly from Maxwell's equations) and also it is quite fast relative to other full-wave analysis methods, such as FDTD, for example. For maximum efficiency of this combination of two methods, we use specific surface filters. The paper contents information of physical sense of these filters and some examples of usage. With the approach, different waveguide parts of electron devices can be synthesized with high efficiency. Calculations of two practical units illustrate the developed method: the waveguide mode converter with operating frequency 10 GHz and waveguide radius R_W = 30 mm, which transforms $TM_{0,1}$ mode into $TE_{0,1}$, and gyrotron launcher with operating frequency 60 GHz and input radius R_W = 15.5 mm, which converts $TE_{7,3}$ mode of the circular waveguide into the inclined Gaussian wavebeam.

Keywords: efficient synthesis method, MLFMA, wave guiding components, electron devices.

1. Introduction and background

Multimode waveguides are widely used in applications that require high-power and high-frequency microwave transmission, such as plasma heating and technological applications [1, 2]. They are also used in many types of electron devices, such as gyrotron or gyro-TWT, which require components that convert one given field distribution into another or preserve the specified field distribution, when the transmission line is bent or some of its parameters (e.g. radius) are changed. Finding of a waveguide wall profile that ensures the desired field transformation is an inverse-scattering problem and, like many other problems of its kind, it does not have solutions, which are satisfactory in all aspects. A number of synthesis methods were developed and utilized even in our institute (for example, Scalar Integral Equation and analytical method for quasi-optical applications [3, 4] and the particle-swarm algorithm [5]), but they have some weak points, such as the narrow field of application or low convergence of synthesis procedure (the thousands of iteration are needed).

In this work, a fast iterative procedure for finding a waveguide shape that does the desired field transformation is presented. It takes two calculations of field distributions (from input to output and backward) per iteration and gives optimal deformation value. The number of iterations is about several tens and convergence of the procedure is guaranteed. The method uses EFIE (Electric Field Integral Equation) formulation as analysis of the fields in the waveguide, and can be easily expressed in terms of surface currents calculated with EFIE approach. The integral equation is derived directly from Maxwell's equations, and has wide range of applicability. Unlike the previous reports describing this approach ([7]), this paper contents full analysis of using surface filters, which let increase the convergence of whole approach and improve the final profile of device.

2.1 Method of Synthesis

The method of synthesis is based on the general principles described in [6].

We will consider a waveguide unit with the given input cross-section S_1, output cross-section S_2, electric and magnetic fields at the both cross-sections (E_1 and H_1 at the input, and E_2 and H_2 at the output, respectively). The initial waveguide surface is denoted S_0 and the time dependence is taken as $\exp(-i\omega t)$. Two field distributions are calculated inside the waveguide by some field analysis method. The first one is calculated by using the input field and the second one, by using the spatially reversed output field ($\vec{E}_{-2} = \vec{E}_2^*$, $\vec{H}_{-2} = -\vec{H}_2^*$, the asterisk stands for complex conjugation) as a boundary condition. Assuming that the conversion is complete, these two field distributions are equal to each other or differ only by the constant phase multiplier $\exp(i\varphi_0)$.

The key parameter of convergence of the iterative procedure is the conversion efficiency of the unit in each iteration:

$$\int_{S_2} \left(\left[\vec{E}_1 \times \vec{H}_{-2} \right] - \left[\vec{E}_{-2} \times \vec{H}_1 \right] \right) \vec{n} \, dS = P \, . \qquad (1)$$

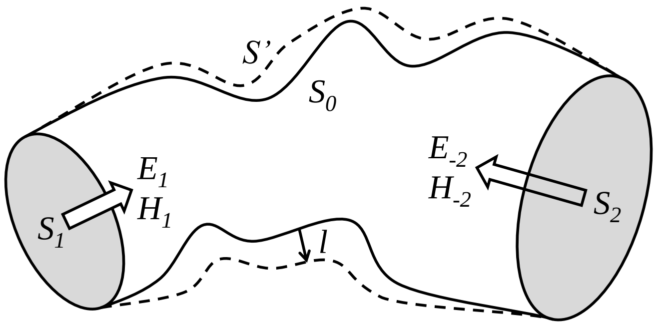

Fig. 1. Explanation of main notations: two given waveguide cross-sections S_1 and S_2 (input and output, respectively), the undeformed waveguide surface S_0 (before iteration) and the deformed surface *S'* (after iteration), the deformation value *l* at some point of the surface, the specified input fields E_1 and H_1, and the desired output fields E_{-2} and H_{-2}. The negative signs in the second pair of the fields mean that they were spatially reversed.

In this terms the main formulas which give the wall deformation *l* in each iteration of the synthesis procedure can written in the next form:

$$l = \alpha \operatorname{Re} F + \beta \operatorname{Im} F, \qquad (2.1)$$

$$F = -ik\left(\vec{H}_{\tau 1} \vec{H}_{\tau -2} + E_{n1} E_{n-2} \right). \qquad (2.2)$$

$$\alpha = \frac{N_{\mathrm{Im}} \operatorname{Re} \Delta P - N_{\mathrm{Re\,Im}} \operatorname{Im} \Delta P}{N_{\mathrm{Re}} N_{\mathrm{Im}} - N_{\mathrm{Re\,Im}}^2},$$

$$\beta = \frac{N_{\mathrm{Re}} \operatorname{Im} \Delta P - N_{\mathrm{Re\,Im}} \operatorname{Re} \Delta P}{N_{\mathrm{Re}} N_{\mathrm{Im}} - N_{\mathrm{Re\,Im}}^2}, \qquad (2.3)$$

$$N_{\mathrm{Re}} = \int_{S_0} \left|\mathrm{Re}\,F\right|^2 dS \quad N_{\mathrm{Im}} = \int_{S_0} \left|\mathrm{Im}\,F\right|^2 dS\,,$$
$$N_{\mathrm{Re\,Im}} = \int_{S_0} \mathrm{Re}\,F\,\mathrm{Im}\,F\,dS. \tag{2.4}$$

where ΔP - is the augment of conversion efficiency after iteration (free parameter).

To fulfill the restriction, the iteration is performed by making the following steps:

1) Calculation of two field distributions inside the undeformed waveguide.
2) Calculation of the function F and the conversion efficiency P_0.
3) Setting the direction (i.e., the phase) of the free parameter ΔP. Absolute value of ΔP is set to any convenient value (e.g. such that P=1 after the iteration).
4) Calculation of l for selected ΔP.

2.2 EFIE formulation

The synthesis procedure may use any field analysis method. In this paper we propose the use of EFIE (Electric Field Integral Equation) solver [7-11].

EFIE is written as:

$$\frac{4\pi}{k}\vec{E}^{\,i}(\vec{r}) = \int_{S_0} (\hat{I} - \frac{\nabla\nabla'}{k^2}) G(\vec{r},\vec{r}\,')\,\vec{j}(\vec{r}\,')\,dS', \tag{2.5}$$

where $G(\vec{r},\vec{r}\,') = \frac{e^{ik(\vec{r}-\vec{r}\,')}}{\left|\vec{r}-\vec{r}\,'\right|}$ - is the Green function for Helmholz equation, $\vec{j}(\vec{r}\,')$ is the unknown surface current, and $\vec{E}^{\,i}(\vec{r})$ is the incident waveguide source field, which is calculated as the radiation of the electric and magnetic surface currents on the input surface S_1 (or output S_{-2}).

The Method of Moments (MoM, [8]) is used to transform this integral equation into a system of linear algebraic equations (SLAE). The so-called RWG-basis [12] is found to be the most suitable for use in conjunction with EFIE-MoM.

The matrix equation can be solved directly by inversion of the matrix Z. This approach requires $O(N^3)$ operations and $O(N^2)$ memory, where N is the number of unknowns, and it can be used for solving problem with N being comparable or less than 10^3. Alternatively, an iterative method to solve the SLAE [8] can be used. It needs $O(N^2)$ operations for the matrix-vector product. Fast Multipole Method (FMM) and its refinement Multilevel Fast Multipole Algorithm (MLFMA) [8-10] need respectively $O(N^{3/2})$ and $O(N \log N)$ memory and operations for matrix-vector product. Nowadays the usage of MoM with MLFMA allows solving EFIE for large multimode systems (with dimentions up to hundreds of the wavelengths).

Usage of EFIE formulation with synthesis procedure described above has many advantages. Firstly, EFIE is a full wave analysis method, which takes into account all waveguide modes automatically. Secondly, it allows calculation of the electromagnetic field in the waveguiding systems with the different types of geometry (circular or rectangular waveguide, round or spiral output cut etc.).

It is important and convenient that in the case of joint use the synthesis procedure and the EFIE calculation the both methods are formulated in the same terms. Really, the synthesis uses the fields on the waveguide wall that are simply rewritten in terms of surface current directly calculated with EFIE approach:

$$\left|\vec{H}_\tau\right| \propto \left|\vec{j}\right|, \;\; E_n \propto div(\vec{j}) .$$

Despite of the high accuracy of EFIE method, some numerical errors can grow with the number of iterations in the synthesis process. Firstly, surface current distribution has artefacts of MLFMA (tree structure) decomposition [7], which are maximal around the group’s edges. Secondly, the field reflected on the open cut of waveguide (as it is in the synthesis of gyrotron launcher) can give additive interference of field distribution, although the reflection on the final profile is rather low. To minimize this effect we use surface filters, which are based on Fourier filter, for smoothing of the synthesized waveguide wall and canceling parasite reflections. Usually, we cut off the part of surface spectrum, which respects to the reflected wave. We can also ignore the perturbation with the scale smaller than the wavelength, which are mostly caused by numerical inaccuracies, not the physical nature. In some cases the filters may even increase the rate of synthesis procedure convergence. We can use additional specific filters to simplify the manufacture of the devices (for example, limit the radius of curvature of the synthesized waveguide wall).

3 Synthesized Units

A number of waveguide mode converters with high efficiency were synthesized with the method described above. In this paper we review two interesting cases of using the approach.

The waveguide mode converter with operating frequency 10 GHz and waveguide radius R_W = 30 mm, which transforms $TM_{0,1}$ mode into $TE_{0,1}$ was synthesized. The method could not give any result starting with regular waveguide wall, because the product of the fields in (2.2) would give a zero, so we had to take a starting shape which partially converts $TM_{0,1}$ to some asymmetric waveguide mode, for example, the rotating $TE_{4,1}$ mode. Then the method can converge successfully and the efficiency of resulting device exceeds 99% (Fig. 2). The maximum of the wall deformation was 5 mm.

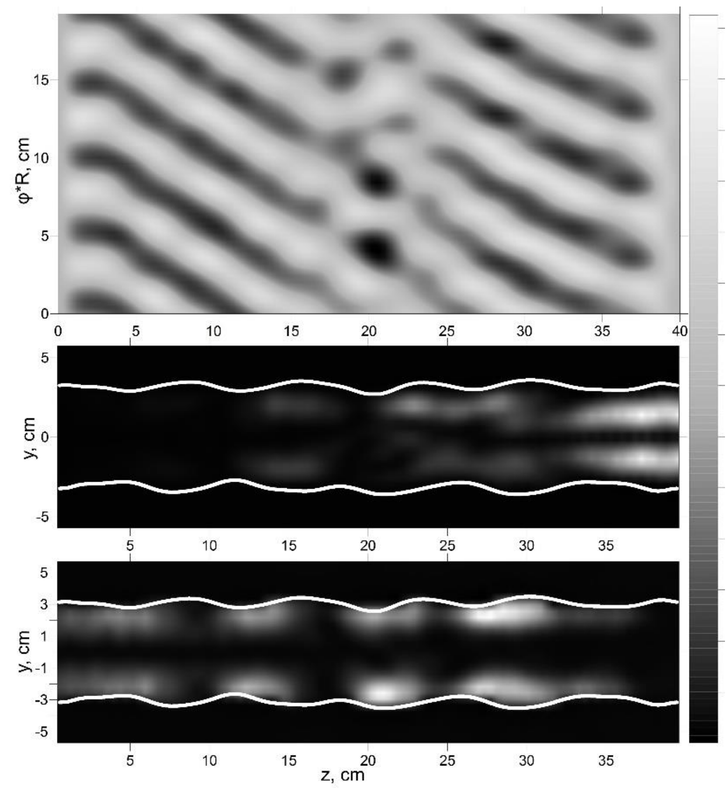

Fig. 2. Synthesized wall deformation of TM_{01} to TE_{01} mode converter (a), and field $|E_x|$ (b) and $|E_y|$ (c) distribution picturing on longitudinal section x=0.

Another interesting case is a gyrotron launcher with operating frequency 60 GHz and input radius R_W = 15.5 mm, which converts $TE_{7,3}$ mode of the circular waveguide into the inclined Gaussian wavebeam. Unlike the geometry of system illustrated on Fig.1 the radiator has spiral output cut. According to synthesis formulation, output surface S_2 must close the total surface (S_1+S_0+S_2). One way to do that is to extend the cylindrical waveguide wall to circular edge and to add the circular surface (for example Fig.3b). For this case it is practical to define current sources for reversed wave on Brillouin's area [13] (dashed line on the Fig 3b) on cylindrical surface continuing unperturbed wall of the converter. The simulations also show that the synthesis of gyrotron launcher works even with open total surface (Fig. 3c). The convergence of synthesis occurs when the S_2 contains the Brillouin area. That makes the analysis of system easier and gives a possibility to use the method of synthesis for constructing open transmission line components. To make a more general statement, in the case of open joint surface we can add virtual patches to input and output and close the surface. For most types of practically interesting geometry the field only leaks outside through these patches. This affects the value of *P*, and the upper estimate of the absolute value of the error is the square root the product of leaked powers. However, if input and reversed output fields mostly leak in different places then these patches have nearly zero influence on *P* and can be neglected in real calculations even if the estimate is not that small.

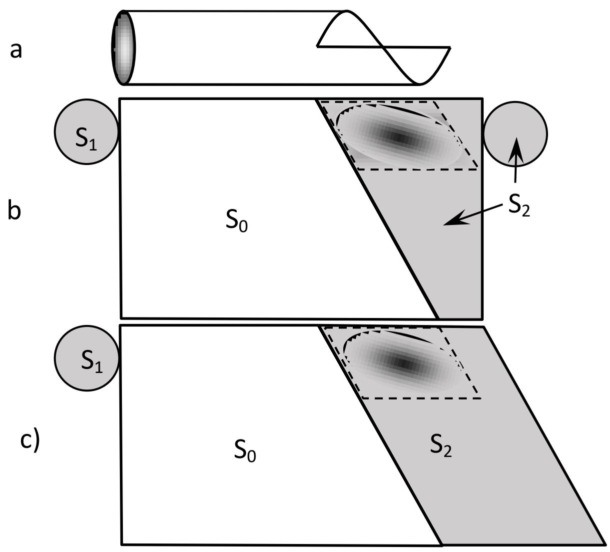

Fig.3 Schematic view of gyrotron launcher (a); input and unrolled wall surfaces with the output surface S_2 closing the total surface (S_1+S_0+S_2) of system (b); example of output surface, which is not closing the total surface (c).

For the gyrotron launcher the required output field distribution is modeled with target function

$$\cos^b(x)\cdot\cos^b(y),$$
$$1 < b \le 2,$$
$$x = (z + \frac{l_B}{2\pi}\varphi - Z_B)\frac{\pi}{l_B}, \qquad (3)$$
$$y = (\varphi - \varphi_B)\frac{\pi}{\Delta\varphi_B},$$

where l_B , $\Delta\varphi_B$, Z_B and φ_B are sizes and coordinates of the center of the Brillouin's area in both directions (z and φ). The function defined on the Brillouin area. The field intensity in the center of the output beam increases with the rate b (this can cause excessive heat in the device), so we chose it equal to 1.5. The target function is rather close to Gaussian function (Gaussian content is about 99.7%) and it is smooth enough to consider all the fields equal to zero on the edge of the launcher cut.

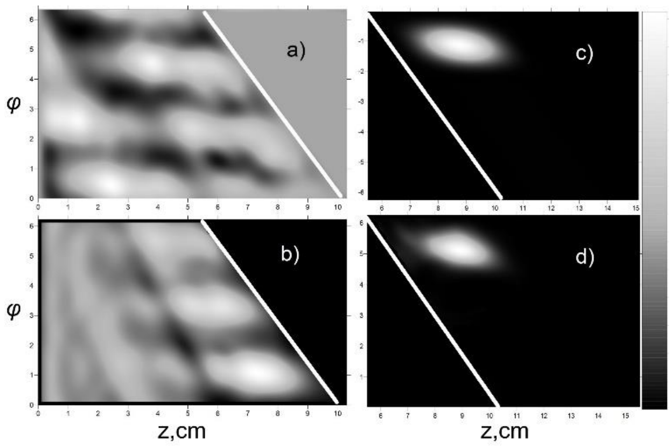

Fig. 4. Profile of gyrotron launcher synthesized with the new approach (a); calculated amplitude of the field ($|H_z|$) on the waveguide wall (b); amplitudes of the target field distribution ($|H_z|$) (c) and resulting field distribution (d) in the output of the launcher

In this case, we had to use the surface filters mentioned in III during synthesis to exclude the wave reflected from open output of the launcher. Calculated efficiency over 99% was reached after three iterations only (Fig. 4). The Cross-polarization of output radiation was about 0.25% and the maximum of the wall deformation was 0.3 mm.

4. Conclusion

The developed iterative synthesis method based on field calculation by EFIE proved to be fast and efficient and showed itself as a powerful tool for designing passive units of vacuum electron devices. It can be used for precise synthesis of efficient waveguide components of 3D shape or for improvement of existing designs made with less precise methods.

Acknowledgments

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