A TECHNIQUE FOR OBTAINING
A HIGH-PRECISION HEURISTIC SOLUTION
TO THE PROBLEM OF DIFFRACTION ON A HALF-PLANE
WITH NON-IDEAL BOUNDARY CONDITIONS

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Abstract. A new technique for obtaining a precise heuristic formula for solving the problem of diffraction of an electromagnetic wave on a half-plane with non-ideal boundary conditions employing the recently developed method of fundamental components is proposed. As a primary heuristic formula, we take a combination of the reflection and transmission coefficients for an infinite plane with non-ideal boundary conditions and the known expression for the diffraction coefficient of a perfectly conducting half-plane. The technique for refining the primary heuristic formula represents a linear combination of two types of heuristic formulas in order to zero the scattering pattern at the margins while retaining the previously found values at the singularity points. The effectiveness of this method of refining is illustrated by solving the problem of diffraction of an electromagnetic wave on a half-plane with generalized two-sided impedance boundary conditions.

Key words: boundary value problems, electromagnetic diffraction, heuristic approaches in diffraction theory, impedance boundary conditions, physical theory of diffraction, surface impedance.

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Introduction

Analytical solutions of diffraction theory play an important role in solving various problems of practical interest in science and technology. Examples of such practical problems include scattering on targets with reduced radar visibility, propagation of electromagnetic waves in urban environment, light scattering on photodetector matrices, etc.

Rigorous analytical solutions are complex and mainly concern structures, geometry of which allows separation of variables. Analytical formulas increase the efficiency of solving practical problems. However, it is very risky to expect that you will always be able to obtain the necessary rigorous analytical solution in the required time. Obtaining some important rigorous analytical solutions took a long time. For example, solving the problem of diffraction on a plane perfectly conducting angular sector took more than 100 years [1]. As a result, heuristic approaches were again employed to solve topical diffraction problems.

In contrast to rigorous analytical and numerical approaches based on mathematical principles, heuristic methods [1–3] are based to a greater extent on physical principles and knowledge of the properties of the diffraction process. However, heuristic approaches have lower accuracy compared to numerical methods. For some practical problems, accuracy of the heuristic approaches may be suitable, but for others, it is not.

A) Heuristic approaches

The geometrical optics (GO) method, like the Kirchhoff integral (or, equivalently, the method of physical optics – PO) [2–4], can be applied to a scatterer of any shape, but it cannot be guaranteed that the accuracy will meet the requirements of a practical problem. To clarify these approaches, the geometric theory of diffraction (GTD) method [5–7] and the method of edge waves (MEW) are used. Another name for MEW is the physical theory of diffraction (PTD) [8–10].

Traditional heuristic approaches (THA) include (GO, PO, GTD, PTD (MEW)). As it is known, THA are based on the so-called «postulates» and do not imply further clarification. By a «postulate» we mean an algorithm of actions to obtain a heuristic

solution. They have a built-in methodological error. If a practical task requires greater accuracy than THA allows, other approaches should be employed. Therefore, the development of new heuristic methods that can increase the accuracy of traditional heuristic approaches is of considerable interest.

In their works, Young, Fresnel, Huygens, and Kirchhoff developed heuristic approaches [2, 4]. Since Sommerfeld, diffraction theory has been developed as a boundary value problem of mathematical physics. Then again [11] heuristic approaches were employed, which demonstrated their high efficiency against the background of the complexity of analytical solutions and the resource intensity of numerical methods. The purpose of obtaining heuristic formulas is to employ them in solvers to increase performance, as well as the physical interpretation and verification of numerical solutions.

Heuristic solutions to practical problems on finite-size scatterers are constructed using solutions to the so-called «reference» or «model» problems on semi-infinite scatterers. The possibility of introducing clarifying amendments based on reference problems is based on the principle of field locality established by Fock. The principle is that the field perturbation on a scatterer geometry feature is located in the vicinity of this feature. It follows that on a feature located on a semi-infinite scatterer, the field perturbation is the same as on a feature located on a finite-sized scatterer. By selecting one of the features on a finite-sized scatterer and correcting it using the corresponding reference solution, it is possible to refine the solution for the finite-sized scatterer.

B) Method of fundamental components

This work develops the recently proposed method of fundamental components (MFC) [1, 12], which can be employed to refine heuristic solutions. The condition for using the MFC is to obtain a verification solution, usually numerical. Then, based on the solutions of simpler problems (or a combination of such solutions), a primary heuristic formula is constructed. After that, the calculation results according to the primary heuristic formula are compared with the verification solution, and an adjustment function is constructed based on the difference between these solutions.

The purpose of this function is to combine with the primary heuristic function in order to produce a final heuristic formula that best matches the verification solution.

C) Impedance boundary conditions

Two-sided impedance boundary conditions were considered in [13–17]. Obtaining heuristic formulas for the problem of diffraction on a half-plane with two-sided impedance boundary conditions was considered in a number of works [18–28]. The difference between this article and other works is in the relationship between the parameters *Xe* and *Xo*, as well as in the method for obtaining the heuristic formula.

There are also works on obtaining heuristic formulas for scatterers with another type of boundary conditions, namely, with the Malyuzhinets boundary conditions at the interface between two media [29–35]. They also obtained compact heuristic formulas that can be employed instead of a rigorous solution. But these formulas are based on rigorous analytical solutions. Unlike these works, the MFC approach is suitable for cases where there is no analytical solution, i.e., for numerical solutions, or, for example, for the physical interpretation of experimental results.

1. The problem under study

The geometry of the problem is shown in Fig. 1.

On the symmetric [19] half-plane for $x < 0$, $y = 0$, the generalized two-sided impedance boundary conditions are satisfied:

$$\begin{cases} -Z_1 H_x^+ + Z_2 H_x^- = E_z^+ \\ -Z_2 H_x^+ + Z_1 H_x^- = E_z^- \end{cases} \tag{1}$$

where $Z_{1,2}$ are the impedances of the half-plane, the plus sign corresponds to the fields on its upper surface, and the minus sign corresponds to the lower one. The impedance Z_1 describes the intrinsic properties of the surface, and the impedance Z_2 describes the connection of fields on different sides of the half-plane.

From (1) one can obtain [19] two boundary conditions for even and odd excitation:

$$\begin{cases} -Z_{e,o}H_x^{e,o} = E_z^{e,o} \\ Z_e = (Z_1 + Z_2)/2 \\ Z_o = (Z_1 - Z_2)/2 \end{cases}.$$

(2)

and when a TM-polarized wave interacts with an infinite surface with boundary conditions (1) and (2), the reflection and transmission coefficients R_{TM} and T_{TM} are defined as follows [19]:

$$R_{TM}\left(X_e, X_o, \psi_0\right) = -\frac{1}{2}\left(\frac{1 - \dfrac{Z_o q_0}{kW_0}}{1 + \dfrac{Z_o q_0}{kW_0}} + \frac{1 - \dfrac{Z_e q_0}{kW_0}}{1 + \dfrac{Z_e q_0}{kW_0}}\right),$$

$$T_{TM}\left(X_e, X_o, \psi_0\right) = \frac{1}{2}\left(\frac{1 - \dfrac{Z_o q_0}{kW_0}}{1 + \dfrac{Z_o q_0}{kW_0}} - \frac{1 - \dfrac{Z_e q_0}{kW_0}}{1 + \dfrac{Z_e q_0}{kW_0}}\right).$$

(3)

In (2) and (3) $Z_e = iX_e$, $Z_o = iX_o$, i is the imaginary unit, $W_0 = 120\pi$, $q_0 = k\sin(\psi_0)$, $k = 2\pi/\lambda$, λ is the wavenumber, X_e and X_o are real parameters characterizing the value of the reactive impedances Z_e and Z_o.
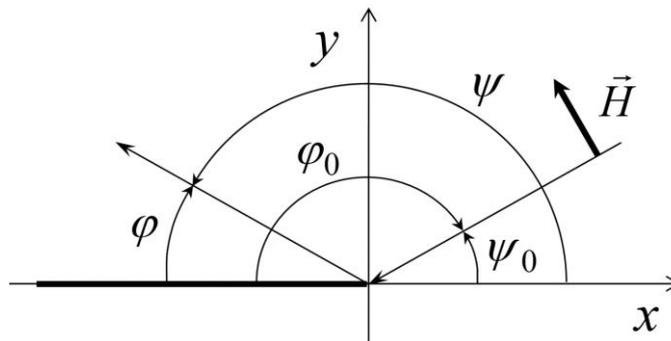

Fig. 1. Geometry of the problem.

We employ two types of coordinate representations (φ, φ_0) and (ψ, ψ_0) in order to avoid confusion in formulas taken from different sources. Note that the following relations hold:

$$\varphi(\psi) = \pi - \psi , \quad \varphi_0(\psi_0) = \pi - \psi_0.$$

(4)

The relationships for R_{TM} and T_{TM} at different Xo and Xe follow from (3).

For the case ($Xo = 0$, $Xe \neq 0$) the following relation holds:

$$T_{TM} = 1 + R_{TM}.\tag{5}$$

For the case ($Xo \neq 0$, $Xe = 0$) the following relation holds:

$$-T_{TM} = 1 + R_{TM}.\tag{6}$$

2. Heuristic formulas for the case ($Xo = 0$, $Xe \neq 0$)

This section presents formulas obtained earlier in works [23–25] for further extraction of individual elements ("fundamental components") from them, which will then be applied in new formulas.

The construction of the MFC formulas is based on the establishing compliance between heuristic formulas and the features of the verification solution. In our case (as will be described in detail later), the features are the value of the diffraction coefficients at the singularity points, as well as at the margins of the scattering pattern.

As a primary heuristic formula, we take a combination of the reflection and transmission coefficients R_{TM} and T_{TM} (3) for an infinite plane with boundary conditions (1) and the known expression for the diffraction coefficient of a perfectly conducting half-plane in the case of TM- polarization, which has the form [1, 8–10, 12]:

$$f(\varphi, \varphi_0) = \frac{1}{2}\left(\frac{1}{-\cos\frac{\varphi-\varphi_0}{2}} - \frac{1}{-\cos\frac{\varphi+\varphi_0}{2}} \right) = \frac{2\sin\frac{\varphi}{2}\sin\frac{\varphi_0}{2}}{\cos\varphi + \cos\varphi_0}.\tag{7}$$

Then the primary heuristic formula [1, 25] is:

$$fg(R_{TM}, T_{TM}, \psi, \psi_0) =$$
$$= \frac{1}{2}\left(\frac{1-T_{TM}}{-\cos\frac{\varphi(\psi)-\varphi_0(\psi_0)}{2}} + \frac{R_{TM}}{-\cos\frac{\varphi(\psi)+\varphi_0(\psi_0)}{2}} \right).\tag{8}$$

Diffraction coefficients (7) and (8) have singularities at the points $\varphi_{s1} = \pi + \varphi_0$ (the "light – shadow" boundary for the transmitted wave) and $\varphi_s = \pi - \varphi_0$ (the "light – shadow" boundary for the reflected wave).

Note that in the case ($Xo = 0$, $Xe \neq 0$) (5) the relation $1-T_{TM} = -R_{TM}$ takes place and thus we have for (8)

$$fg\left[R_{TM}, T_{TM}, \psi(\varphi), \psi_0(\varphi_0)\right] = -R_{TM} f(\varphi, \varphi_0). \tag{9}$$

The case ($Xo \neq 0$, $Xe = 0$) (6) for constructing heuristic formulas is more complicated, relation (5) is not satisfied, since $|1+T_{TM}| = |R_{TM}| \neq |1-T_{TM}|$.

As a verification solution to the problem of diffraction on the impedance half-plane (1) for TM polarization, we employ the rigorous solution according to the Wiener-Hopf method (WHM) [18, 21]:

$$fr\left(X_e, X_o, \psi, \psi_0\right). \tag{10}$$

We will call (10) the «WHM solution». To compare the WHM solution with the primary heuristic formula (8) (in the terminology of the MFC – verification), we will construct the function $fgr(R_{TM}, T_{TM}, \psi, \psi_0)$, which we will call the «comparison function» for fg (8).

$$fgr\left(R_{TM}, T_{TM}, \psi, \psi_0\right) = \frac{NTM \cdot fr\left(X_e, X_o, \psi, \psi_0\right)}{fg\left(R_{TM}, T_{TM}, \psi, \psi_0\right)}. \tag{11}$$

The constant factor *NTM* in (11) is a technical normalization between the heuristic formulas and the verification solution, which is used in this work. In our case, there is an analytical expression $NTM = |(2\pi ki)^{0.5}|$. In other cases, the value of the normalizing factor can be determined by setting the parameters of the scatterer, for which there is a strict analytical solution that must coincide with the heuristic one. For example, the combination of parameters ($Xo = 0$, $Xe = 0$) leads to the fact that $R_{TM} = -1$, $T_{TM} = 0$, i.e. the impedance half-plane becomes perfectly conducting. So "non-ideal boundary conditions" turn into "ideal" one. Comparing the heuristic solution (7) or (8) with the numerical solution (10), found for ($Xo = 0$, $Xe = 0$), we determine the presence of the normalizing factor and its value.

If the primary heuristic formula fg (8) accurately described the strict solution according to the WHM solution (10), then the function $fgr(R_{TM}, T_{TM}, \psi, \psi_0)$ (11) would be equal to 1 or close to this value. But at the first stage this is not the case.

In accordance with the MFC, for the case ($Xo \neq 0$, $Xe = 0$) we will search the adjustment function, which in combination with the primary heuristic formula will

bring the heuristic solution closer to the verification solution (10). In this case, we will obtain the «final» heuristic formula.

For the case ($Xo = 0$, $Xe \neq 0$) (5), a good match was obtained [23 – 25] between the final heuristic formula and the verification solution. In this case, the final heuristic formula *FHE* has the form

$$
\begin{aligned}
FHE(m_{TM}, X_e, X_o, \psi_0, \psi) = \\
= \frac{fg(R_{TM}, T_{TM}, \psi, \psi_0)}{NTM} \frac{cx_{TM}(m_{TM}, X_e, X_o, \psi)}{cx_{TM}(m_{TM}, X_e, X_o, \psi_s)}.
\end{aligned} \tag{12}
$$

and the non-normalized (by the value at point ψ_s) adjustment function cx_{TH} turned out to be like this

$$
\begin{aligned}
cx_{TM}(m_{TM}, X_e, X_o, \psi) = \\
= R_{TM}\left[m_{TM}(X_e) \cdot X_e, m_{TM}(X_o) \cdot X_o, \frac{\psi}{2} \right],
\end{aligned} \tag{13}
$$

where

$$
m_{TM}(X) = \sqrt{1/2\left[1 + \sqrt{1 + 1/4(W_0/X)^2}\right]}. \tag{14}
$$

In [23 – 25], to obtain the *FHE* function (12), in the first stage the function *fgr* (11) was investigated. It was found that its shape is similar to R_{TM}, but the periodicity of *fgr* with respect to the angular coordinate ψ is different. Taking this feature into account, the function (13) was constructed in the second stage, which included the unknown function (14). In the third stage, the function (14) was found.

3. Generalizing formula for R_{TM}

For the case ($Xo \neq 0$, $Xe = 0$) we employ the same adjustment function cx_{TM} (13) as for ($Xo = 0$, $Xe \neq 0$). It is possible to construct a generalizing formula RR_{TM}, combining the reflection coefficient R_{TM} (3) and the adjustment function cx_{TH} (13), normalized to its value at the singularity point ψ_s

$$
\begin{aligned}
RR_{TM}(m_{TM}, X_e, X_o, \psi_0, \psi, \psi_s) = \\
= R_{TM}(X_e, X_o, \psi_0) \frac{cx_{TM}(m_{TM}, X_e, X_o, \psi)}{cx_{TM}(m_{TM}, X_e, X_o, \psi_s)}.
\end{aligned} \tag{15}
$$

Formula (15) includes the singularity point ψ_s associated with the reflection angle: $\psi_s = \pi - \psi_0$. When substituting the singularity point ψ_{s1} associated with the "straight forward" angle, we get: $\psi_{s1} = \psi_0 - \pi$.

As follows from formula (13), the cx_{TH} adjustment function is an expression for R_{TM} (3) with the angle ψ_0 replaced by ψ and the period of cx_{TM} variation by angle ψ doubling compared to the period of $R_{TM}(\psi_0)$ variation by angle ψ_0. In addition, the m_{TM} multiplier is employed to change the dependence of R_{TM} on Xe and Xo in cx_{TM}. Normalized to its value at the singularity points, the adjustment function cx_{TM} at these points is equal to 1, and the values of $RR_{TM}(\psi_s)$ and $RR_{TM}(\psi_{s1})$ coincide with $R_{TM}(\psi_0)$. For other values of the angle ψ, the functions RR_{TM} and R_{TM} generally differ.

For compactness, we will further write the left-hand side of (15) as $RR_{TM}(\psi_s)$ and $RR_{TM}(\psi_{s1})$. Note also that the function R_{TM} depends only on the angle ψ_0, and does not depend on ψ (unlike RR_{TM}, which depends on both ψ and ψ_0).

4. Heuristic formulas for the case ($Xe = 0$, $Xo \neq 0$)

In this section, we will derive new heuristic formulas employing the previously obtained "fundamental component", namely RR_{TM}.

At the first stage of the adjustment process, we find the factors (numerators) at the singularities. Let us denote as $fr(\varphi_{s1})$ and $fr(\varphi_s)$ the values of the verification function fr (10) at the singularity points φ_{s1} and φ_s, which correspond to the scattering angles of the reflected and transmitted waves.

Comparison of formulas (3) and (11) at the singularity points φ_{s1} and φ_s at $Xe = 0$ showed that for all values of Xo the following relations hold:

$$
\begin{aligned}
NTM \cdot fr\left[X_e, X_o, \varphi_{s1}(\psi_{s1}), \varphi_0\right] &\cong 1 - i - iR_{TM}, \\
NTM \cdot fr\left[X_e, X_o, \varphi_s(\psi_s), \varphi_0\right] &\cong 1 + i + iR_{TM}.
\end{aligned}
\tag{16}
$$

Based on (3), (15) and (16), we introduce two types of heuristic diffraction coefficients (primary heuristic formulas) $FH1$ and $FH2$:

$$FH1\big(m_{TM},X_e,X_o,\psi_0,\psi\big)=$$

$$=\frac{1}{2\cdot NTM}\left[\frac{1-i-iR_{TM}}{-\cos\frac{\varphi(\psi)-\varphi_0(\psi_0)}{2}}-\frac{1+i+iR_{TM}}{-\cos\frac{\varphi(\psi)+\varphi_0(\psi_0)}{2}}\right],\qquad(17)$$

$$FH2\big(m_{TM},X_e,X_o,\psi_0,\psi\big)=$$

$$=\frac{1}{2\cdot NTM}\left[\frac{1-i-iRR_{TM}\big(\varphi_{s1}\big)}{-\cos\frac{\varphi(\psi)-\varphi_0(\psi_0)}{2}}-\frac{1+i+iRR_{TM}\big(\varphi_s\big)}{-\cos\frac{\varphi(\psi)+\varphi_0(\psi_0)}{2}}\right].\qquad(18)$$

Here R_{TM} and RR_{TM} are defined by expressions (3) and (15), respectively.

Formulas $FH1$ (17) and $FH2$ (18) are constructed by analogy with (8) and (12), taking into account that the values of $FH1$ and $FH2$ at the singularity points φ_{s1} and φ_s are known (16).

At the points ψ_{s1} and ψ_s, the values of expressions $FH1$ (17) and $FH2$ (18), up to the factor NTM, coincide with the values of the verification function $fr(X_e, X_o, \psi, \psi_0)$ (10). The scattering patterns of the verification function (10) are also obtained for the case $(Xe = 0, Xo \neq 0)$.

Taking the factor out of the brackets, as was done with R_{TM} in (9), is now impossible, since the values of the scattering patterns $FH1$ and $FH2$ at the singularity points φ_{s1} and φ_s differ.

Let us compare the graphs $FH1$ (17) and $FH2$ (18) with the verification WHM solution fr (10) (Fig. 2).

The graphs in Fig. 2 show the calculation results using the primary heuristic formulas $FH1$ (17) and $FH2$ (18). Due to (16), the scattering patterns for $FH1$ and $FH2$ are asymmetric, so, unlike (8) and (12), the function $f(\varphi, \varphi_0)$ cannot be factored out of (17) and (18).
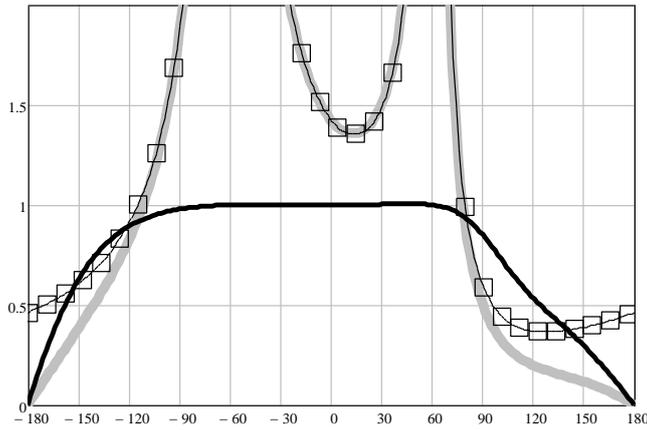
Fig. 2 (a). Scattering patterns for $Xo = 300$, $\psi_0 = 120°$. The angle ψ is plotted horizontally, and the values of the functions are plotted vertically. Fig. 2 (a) shows: the verification function (the WHM solution) fr (10) (solid gray line), the primary heuristic formula $FH1$ (17) (squares), the graph of the "comparison function" fgr (11) for $FH1$ (solid black line).
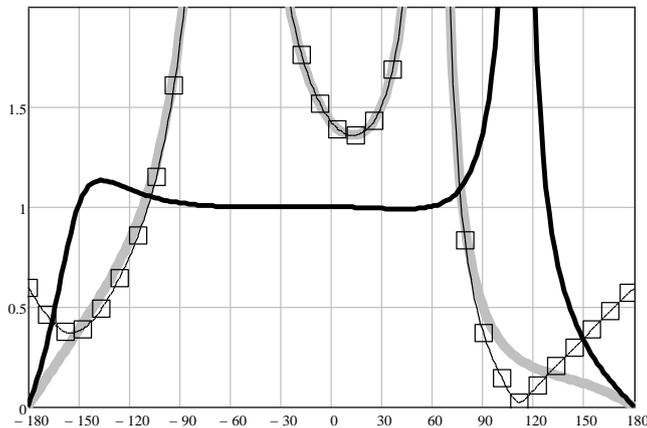

Fig. 2 (b). Scattering patterns for $Xo = 300$, $\psi_0 = 120°$. The angle ψ is plotted horizontally, and the values of the functions are plotted vertically. Fig. 2 (b) shows: verification function (WHM solution) fr (10) (solid gray line), primary heuristic formula $FH2$ (18) (squares), graph of the "comparison function" fgr (11) for $FH2$ (solid black line).

At the points ψ_{s1} and ψ_s, the «comparison function» fgr is equal to 1 for both $FH1$ and $FH2$. At the edges of the angular range $\psi = \pm180°$, both $FH1$ and $FH2$ differ from the WHM solution fr. However, individually, the values of each function $FH1$ and $FH2$ at the edges of the angular range are equal, which, as will be shown later, allows us to combine $FH1$ and $FH2$ to obtain a more accurate solution.

5. Technique for improving matching

None of the *FH*1 or *FH*2 graphs gives a good match with the verification solution, since, despite the match with the verification solution at the singularity points, the values of *FH*1 and *FH*2 at the margins of the scattering pattern differ from the values of the verification function *fr* (10). To eliminate these differences, we find a heuristic formula that is a linear combination of the *FH*1 and *FH*2 functions in such a way that the values at the singularity points φ_{s1} and φ_s remain the same, and the values at the margins become equal to zero, as in the verification function. We find the coefficient *czn* in such a way that the relation is satisfied at the edges of the range of angles

$$
\begin{aligned}
&FH1 - czn \cdot FH2 = 0, \\
&\mathrm{Re}\big(FH1 - czn \cdot FH2\big) = 0, \\
&\mathrm{Im}\big(FH1 - czn \cdot FH2\big) = 0.
\end{aligned}
\tag{19}
$$

Formula (19) can be divided into two ones, for the real and imaginary parts *FH*1 and *FH*2. We find *czn* in the form of the arithmetic mean of the correction formulas for the real and imaginary components

$$
\begin{aligned}
czn = \frac{1}{2}\Bigg\{ &\frac{\mathrm{Re}\big[FH1\big(m_{TM},X_e,X_o,\psi_0,\psi=\pi\big)\big]}{\mathrm{Re}\big[FH2\big(m_{TM},X_e,X_o,\psi_0,\psi=\pi\big)\big]} + \\
&+ \frac{\mathrm{Im}\big[FH1\big(m_{TM},X_e,X_o,\psi_0,\psi=\pi\big)\big]}{\mathrm{Im}\big[FH2\big(m_{TM},X_e,X_o,\psi_0,\psi=\pi\big)\big]}\Bigg\}.
\end{aligned}
\tag{20}
$$

As a result, for the case $(Xo \neq 0, Xe = 0)$ we obtain the final heuristic formula *FHO*.

$$
\begin{aligned}
FHO\big(m_{TM},X_e,X_o,\psi_0,\psi\big) = \\
= \frac{1}{1-czn}\big\{FH1\big(m_{TM},X_e,X_o,\psi_0,\psi\big) - \\
- czn \cdot FH2\big(m_{TM},X_e,X_o,\psi_0,\psi\big)\big\}
\end{aligned}
\tag{21}
$$

As we can see from the Fig. 3, for the linear combination of scattering patterns (21), the function *fgr*(R_{TM}, T_{TM}, ψ, ψ_0) (11) has improved significantly, since it has leveled out and come as close as possible to 1.
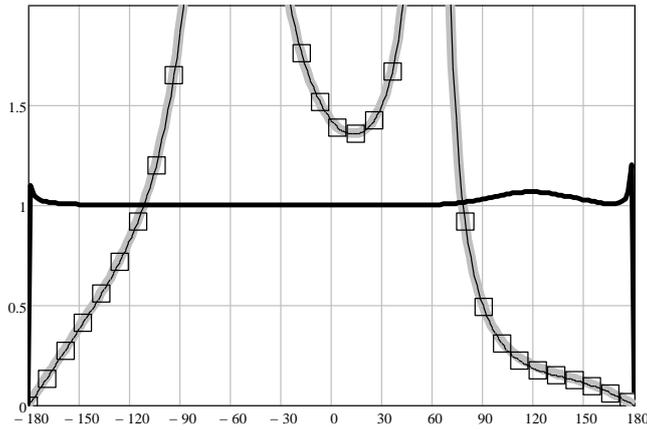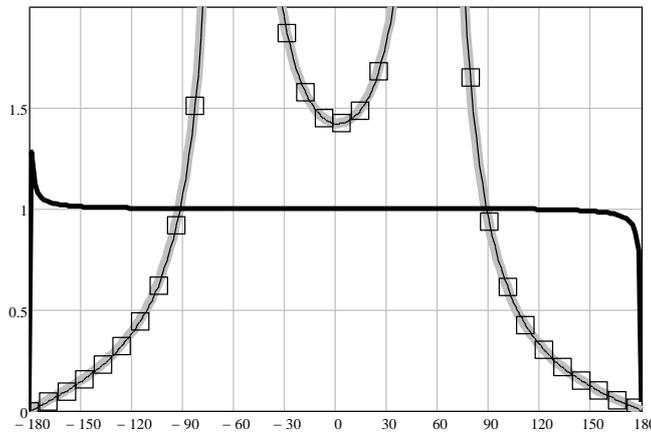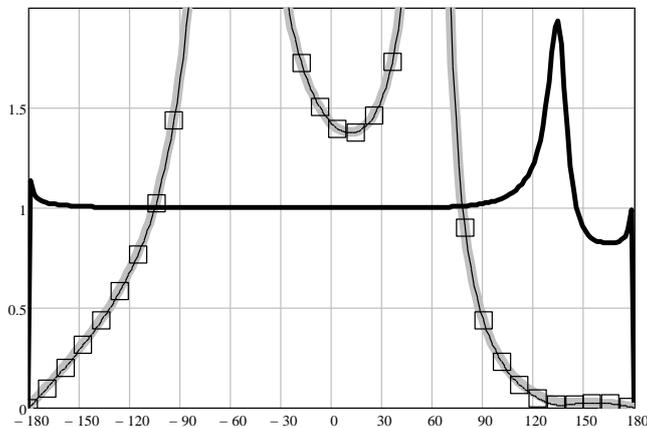
Fig. 3. Scattering pattern for $Xo = 300$, $\psi_0 = 120º$. The angle ψ is plotted horizontally, and the values of the functions are plotted vertically. Fig. 3 shows: verification function (WHM solution) fr (10) (solid gray line), final heuristic formula FHO (21) (squares), graph of the "comparison function" fgr (11) for FHO (solid black line).

Calculations of FHO for some other values of Xo are shown in Fig. 4.

In the Fig. 4 (b), the perturbation process in the right part of the fgr graph is caused by the mismatch of the minima of the heuristic and verification solutions for small values of scattering patterns (SP).
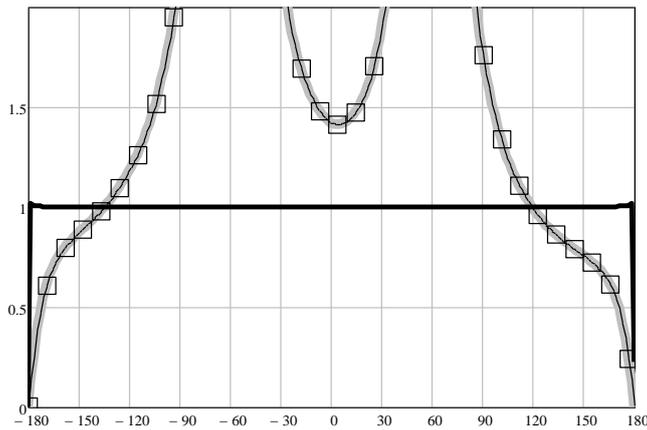

(a) $Xo = 20$

(b) $Xo = 200$

(c) $Xo = 2000$

Fig. 4. Scattering patterns for $Xo = 20$ (a), 200 (b), 2000 (c), $\psi_0 = 120^\circ$. The angle ψ is plotted horizontally, and the values of the functions are plotted vertically.
Fig. 4 (a), (b), (c) show: verification function (WHM solution) fr (10) (solid gray line), final heuristic formula FHO (21) (squares), graph of the "comparison function" fgr (11) for FHO (solid black line).

In fact, as we can see from the SP graph, the difference between the fr and FHO graphs is insignificant.

Due to the limited volume of the article, we present only few calculations, but they show that the proposed method for constructing the final heuristic formula gives the same good agreement for all values of $0 < Xo < \infty$ and $20^\circ < \psi_0 < 160^\circ$.

6. Discussion

In previous works, various techniques for obtaining the adjustment function were proposed to refine the primary heuristic formulas employing the MFC. Among them, we can note the employment of a modified expression for the reflection coefficient and compensation for the slope of the «comparison function». In this work, we found a solution by proposing a new MFC technique which is a linear combination of two types of heuristic formulas in order to zero the scattering pattern at the margins while retaining the previously found values at the singularity points. This technique demonstrated good efficiency in the entire range of angles against the background of the complexity of the problem (the absence of symmetry of the scattering pattern with respect to the central point of the angular range).

The construction of heuristic analytical formulas based on the MFC is similar to the process of obtaining in physics theoretical formulas based on the experimental results. As an «experiment» in the MFC, a verification solution (usually numerical one) is taken, but measurement results can also be employed. Therefore, given the two conditions of application (the verification solution and the primary heuristic formula), the MFC approach can be applied in other areas of physics.

The uniqueness of the MFC is that, unlike many other approaches that require the existence of a solution in the form of analytical expressions, MFC can be employed to refine the primary heuristic formula based on any verification solution: strict analytical, numerical or experimental. The primary heuristic formula can be selected based on general physical concepts of the process under study.

A superficial acquaintance with the material may raise the question: why build heuristic formulas if the solution still needs to be verified?

The answer to this question: we build heuristic formulas for reference (or model) solutions on semi-infinite scatterers. Then we employ these formulas for scatterers of finite size. In this case, verification is also required, but only at the stage of creating heuristic formulas. In this case (as for traditional heuristic methods), we determine the range of angles in which this solution is most accurate. Then the heuristic formulas can be employed without verification, and the larger the size of the scatterer, the more accurate the result will be.

Conclusion

A technique for refining heuristic formulas for the problem of diffraction on a half-plane with non-ideal boundary conditions using the method of fundamental components (MFC) is proposed. The technique is based on taking into account the values of diffraction coefficients at singularity points and at the edges of the angular range. The calculation results employing the example of solving the problem of diffraction of an electromagnetic wave on a half-plane with generalized two-sided impedance boundary conditions demonstrated the effectiveness of the proposed approach.

This article differs significantly from earlier studies, both in the problem formulation and in the solution method. The difference in the problem formulation lies in a new combination of input parameters for the verification solution ($Xo \neq 0$, $Xe = 0$), which leads to an asymmetry in the scattering pattern. The solution to this more complex case differs in a new method for constructing the primary heuristic functions (based on binding to values at the singularity points) and the use of a new adjustment technique (based on binding to values at the edges of the angular range and subsequent linear combination of the FH1 and FH2 functions).

The MFC allows one to construct heuristic formulas of increased accuracy. These formulas can be employed in practical problems instead of numerical solutions. In addition, heuristic formulas can be used for the physical interpretation of numerical solutions, as well as for the verification of numerical solutions in the process of their creation.

Given a verification solution and a primary heuristic formula, the MFC can be applied to other areas of physics. The primary heuristic formula can be a combination of analytical solutions to problems from the same area of physics in their simplest formulation (in our case, it is a perfectly conducting half-plane and reflection from an unbounded plane with the same boundary conditions on the upper and lower surfaces).

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